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Estimation of viscosity contrast and finite strain from deformed elliptical inclusions

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Abstract—A method of strain analysis is described which takes into account a possible competence difference between the strain markers and the rest of the rock. Using data consisting of the sectional shapes and orientations of groups of inclusions from conglomerate-like rocks, limits are placed on the possible effective viscosity contrast between the inclusions and the rock as a whole as well as on the bulk finite strain suffered by the rock. These results are calculated using a computer program, based on the theory for the deformation of elliptical cylinders. The method can be considered an extension of the Rf/ϕ method, but provides more information and involves less restrictive assumptions.

INTRODUCTION

THE SHAPES of pebbles and other sedimentary clasts have been widely used as indicators of tectonic strain. The calculation of the strain has usually been based on the assumption that the internal strain within the inclusion is identical to that of the host rock implying that the inclusions have been transformed in a passive fashion along with the matrix. Although geologists have long been aware of the likely existence of competence differences between inclusion and matrix in such rocks, taking account of this additional variable alters the strain analysis from a problem of purely geometrical character to a physical one of considerable complexity. In pioneering work towards the solution of this problem, Gay (1968a) considered both inclusion and matrix as viscous fluids of differing viscosity. For specific deformation histories Gay derived equations relating the strain of the inclusion to that of the whole rock (the bulk strain) assuming widely spaced pebbles of negligible volume compared with that of the matrix. A finite concentration of particles like that present in conglomerates, Gay suggested, would reduce the influence of the particle/matrix viscosity ratio in the equations describing the relative deformation of the inclusion and the system. He suggested how the 'effective viscosity contrast' of such systems can be calculated from the true viscosity ratio and noted that it decreases rapidly with increase in particle concentration.

For conglomerates composed of pebbles of one type, Gay (1968b) and Gay *et al.* (1976) described a practical method whereby the bulk strain can be estimated from pebble shapes. According to his method the viscosity ratio of matrix and pebbles required for the calculation is not determined *in situ*. Instead a value is taken from compiled viscosity ratios determined from previous analyses of deformed polymict conglomerates where pebbles, corresponding in lithology to that of the matrix and to the pebbles concerned, were present. Assigning a viscosity ratio in this way to a pair of rock-types could be a significant source of error in the analysis of bulk strain. Some recent analyses of conglomerates (e.g. Evans *et al.* 1980, Lisle & Savage 1983) indicate that the competence variations between different pebbles is sometimes more closely related to textural characteristics such as grain size than to compositional variables. These studies suggest that viscosity contrasts between pebbles of two lithological types could vary considerably depending on the deformation mechanisms, metamorphic conditions prevailing at the time of deformation, and so on.

This paper describes how shape and orientation data from inclusions within deformed conglomerates and similar rocks can be used to estimate the viscosity ratio and the finite strain suffered by the conglomerate, obviating the need to assign a viscosity contrast on the basis of previous analyses elsewhere. We begin with a description of the proposed method developed by one of us (R.J.L.) and proceed by describing its application to field examples including conglomerates studied by the other authors in the Swedish Grythyttan area.

THE THEORETICAL BASIS OF THE METHOD

The method described below can be considered as an extension of the Rf/ϕ technique for the analysis of tectonic strain from elliptical markers. The axial ratios (Rf) and orientations (ϕ) of deformed elliptical markers as seen on planar sections show variations which can be attributed to variations in pre-tectonic shape and orientation. Ramsay (1967) and Dunnet (1969) decribe how the pattern of the plotted markers on an Rf/ϕ diagram, assuming the absence of initial preferred orientations, can be used to deduce the tectonic strain ratio on planar outcrops. Dunnet & Siddans (1972) and Matthews *et al.* (1974) modified the Rf/ϕ technique to make it able to handle sedimentary fabrics defined by pre-tectonic preferred orientations of the elliptical markers.

The existence of competence contrasts and the resulting state of heterogeneous strain creates problems when determining the strain on planar sections through the rock. A planar section through the deformed rock will contain material points which were originally not situated in this plane, as a consequence of the differential body rotations of the matrix and pebbles. This means that the deformed geometry of the planar section is not solely dependent on the strain ellipse on that plane but is also affected by the three-dimensional strain. It thus appears that the strain determination is a truly three-dimensional problem and that the strain ellipsoid cannot be built up from strain ellipses on several planes.

There are two restrictions which prevent us treating the problem in a three-dimensional fashion: firstly, there is a lack of theory appropriate to the deformation of inclusions with a general orientation and triaxial ellipsoidal shape (Gay *et al.* 1976) and secondly, three-dimensional shape and orientation data on inclusions is more difficult to obtain and represent.

Because of this we make a simplification which allows the application of a two-dimensional method of analysis. We treat the outlines of the inclusions as belonging to elliptical cylinders whose axes are perpendicular to the planar section being considered. This model does not involve rotation of the markers with respect to the considered section, so that the consequences of this simplification will become increasingly serious with increasing competence contrast between the ellipsoidal inclusion and the matrix.

The equations presented by Bilby & Kolbuszewski (1977), describing the pure shear deformation of an viscous elliptical cylinder in terms of its orientation, its axial ratio and the viscosity ratio between cylinder and matrix have been applied in the analysis (Fig. 1). The equations expressed in terms of the variables commonly used in Rf/ϕ analyses (e.g. Dunnet 1969) are:

$$\ln Rs = 2\int_{Ri}^{Rf} \frac{J(R)G(R)}{2R(R+1)^2[G^2(R) - G^2(Ri)\sin^2 2\theta]^{1/2}}\,dR$$

and:

$$\sin 2\phi = \frac{G(Ri)}{G(Rf)}\sin 2\theta$$

where R = axial ratio of the inclusion,
Ri = initial axial ratio of the inclusion,
Rf = final axial ratio of the inclusion,
Rs = bulk strain ratio $(\lambda_1/\lambda_2)^{1/2}$,
θ = initial orientation } with respect to finite maximum elongation direction
ϕ = final orientation } with respect to finite maximum elongation direction

and

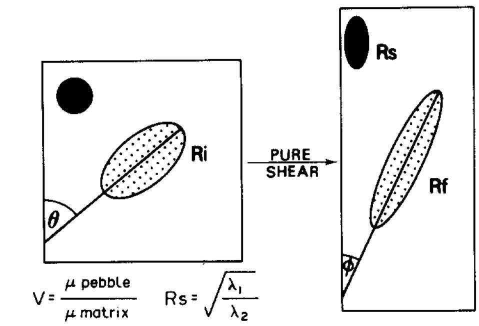

Fig. 1. The teminology for this Rf/ϕ method.

$$J(R) = R^2 + 2VR + 1$$

$$G(R) = \frac{R^2-1}{R}\left[\frac{VR^2 + 2R + V}{(R+1)^2}\right]^{v}$$

where V = viscosity ratio, $\mu_{\text{pebble}}/\mu_{\text{rock}}$. These equations allow the calculations of final orientation and axial ratio of an elliptical inclusion whose initial orientation and axial ratio is known, together with shape and orientation of the strain ellipse. They can, if necessary, be used to calculate the theoretical Rf/ϕ curves for different strains and different viscosity contrast comparable to those used by Dunnet (1969) and Lisle (1977). Such curves have a unique shape for each value of Rs and V. Matching of natural Rf/ϕ patterns with such theoretical reference curves is a tedious task due to the large number of theoretical curves available for comparison. For this reason we have resorted to a form of analysis of Rf/ϕ data which makes use of the computer program described below that is based on reciprocal deformation into the undeformed state.

THE CALCULATION OF VISCOSITY CONTRAST AND FINITE STRAIN

The equations above form the basis of a computer program used to estimate viscosity ratios and finite strain from natural Rf/ϕ data. This program, similar in design to that described by Peach & Lisle (1979), simulates in reverse the deformation of the markers. For a selected value of viscosity contrast, the markers are subjected to a mathematically simulated pure shear deformation whose λ_1 direction is perpendicular to the direction of λ_1 in the considered section through the deformed rock. The latter is assumed to be parallel to the vector mean of the long axes of the deformed markers. The marker orientations which result from this de-straining are then analysed statistically. The orientations of the long axes are grouped into orientation classes and the resulting distribution is compared to a uniform orientation distribution model. This is done by means of Hodges–Ajne's test (Mardia 1972, p. 182), which assesses the probability that the de-strained orientations are a random sample from a uniform distribution. The hypothesis of uniformity is tested on the basis of tests statistic, m^*. The test is appropriate when deviations from uniformity of a cyclic nature are suspected.

The de-straining procedure is repeated several times with changed values of the viscosity ratio and/or reciprocal strain. In this way the statistic m^*, expressing the degree of uniformity of the undeformed marker orientations, can be calculated for a large number of combinations of applied reciprocal strains and viscosity ratios. Assuming, as we have done here, that the pre-tectonic orientations belonged to a uniform distribution, we obtain a number of possible strain ratios and viscosity ratios which in the simulation were able to produce a sufficiently uniform distribution of 'undeformed' long axis orientations. We plan to develop the program further so that initial bedding-parallel symmetrical clast fabrics can also be assumed.

The nature of the results yielded by the program is illustrated by the applications to natural Rf/ϕ data described below.

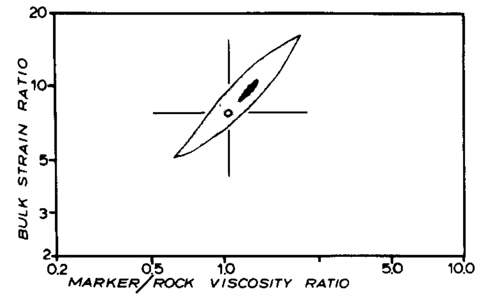

Fig. 2. Results from the computer program showing statistically acceptable bulk strain and viscosity contrasts (the shaded area with $m^* < 2.5$) for the deformed cordierite crystals reported by Robin (1977). The inner contour is at $m^* = 1.5$. The intersection of the large cross represents Robin's estimate; the vertical line shows the assumed viscosity contrast and the horizontal line the calculated strain ratio.

Example 1: Robin's cordierite crystals

The Rf/ϕ data from cordierite crystals in a deformed schist published by Robin (1977, fig. 2) have been analysed by the method outlined above. The results are presented graphically in Fig. 2. This shows contours of m^* as measure of the uniformity of the de-strained markers. The graph illustrates how the m^* yielded by the de-strainings varies as a function of the bulk strain ratio (the reciprocal strain applied at the time of de-straining) and viscosity ratio. Strains and viscosity ratios which yield values of m^* below 2.5 (the 5% significance point) are considered possible solutions and are enclosed by the outer contour. As this contour is centred around $V = 1$ we have no grounds for objecting to Robin's assumption that no viscosity contrast exists. Robin's calculated strain (see Fig. 2) lies within the range of possible bulk strains calculated using our method.

Example 2: The Swedish conglomerate

A second example using data from pebbles in a polymict conglomerate illustrates two points. Firstly it shows that results from some data sets are vague, indicating a rather wide range of possible strains and viscosity contrasts. Secondly it illustrates how, by making measurements on several planes through the same rock, the vagueness of results can be drastically reduced. Relevant geological details about the conglomerate and its setting are presented in the Appendix. The average axial ratios of the various compositional pebble groups are shown in a Flinn diagram (Fig. 3). These differences in average shape together with field observations (Fig. 4) suggested the existence of competence differences between the pebble groups.

For each section through the rock, Rf/ϕ data were collected for each of the pebble groups. Figure 5 shows the results produced by the computer program; the design of these graphs is the same as for Fig. 2 described in the previous section. For some data groups, the results are far from precise in indicating the bulk strain and viscosity ratio. However, by simultaneously considering several planar sections and, as we have been able to do here, by considering markers belonging to several competence groups, we place further constraints on the results. In this conglomerate, for example, on any one plane the calculated bulk strain should be the same whichever pebble group was used. This is indicated by the horizontal lines in Fig. 5. Likewise, for a particular pebble group, the viscosity contrast is expected to be alike for all planes (provided that pebble concentration and lithologies are constant). These are the vertical lines in Fig. 5. Using these constraints on the results, the estimated bulk strain axial ratio for the XZ plane was 6.1 and for the YZ plane 3.0 (Fig. 5). Corresponding viscosity ratios between pebbles and the bulk rock are 9, 2, 1.4, 1.3, 0.9 and 0.85 for the pebbles classified as quartz, hard leptite, soft leptite, hälleflinta, light shale and dark shale, respectively (Fig. 5). These viscosity ratios might be related to the grain size and mineral composition of the rock (see Appendix).

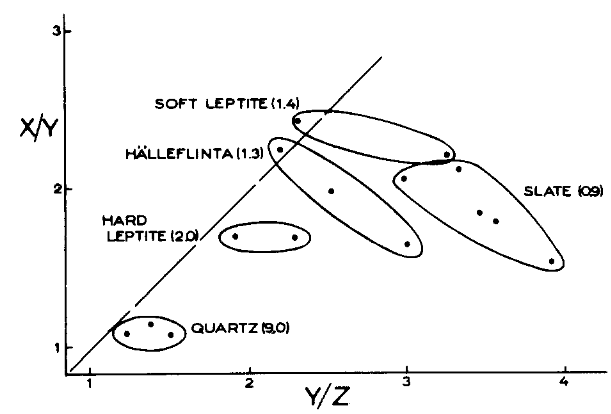

Fig. 3. Pebble shape plotted on a Flinn diagram. Each dot represents the average axial ratio of a specific compositional pebble group within a restricted volume of rock. These averages have been calculated from measurements of between 20 and 270 pebbles on the XZ and YZ-planes. The number in brackets is the viscosity contrast between that pebble type and the conglomerate calculated using the method described in this paper.

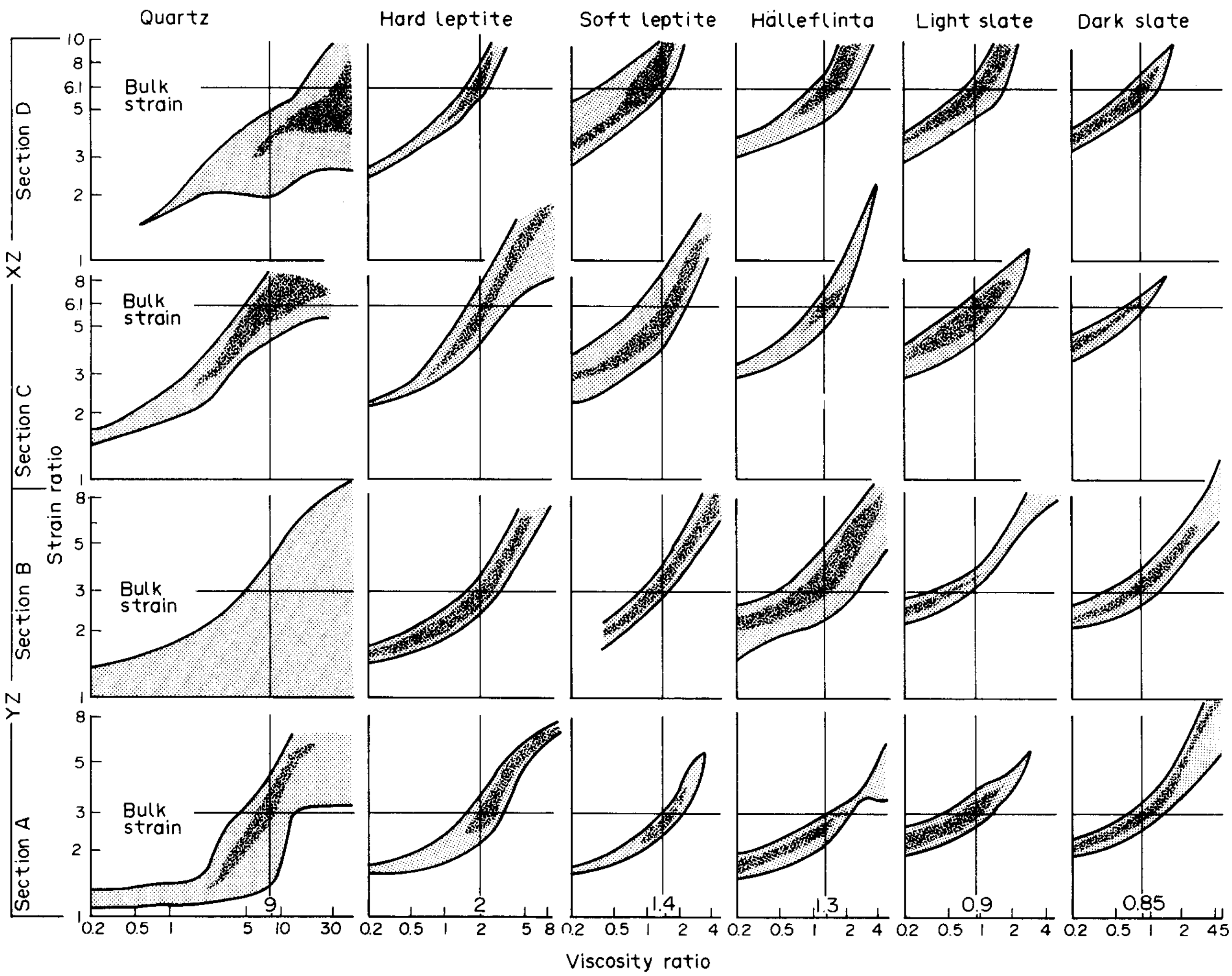

Fig. 5. Swedish conglomerate example; four different sections, six pebble types. Shaded areas denote combinations of viscosity contrast and bulk strain capable of explaining the Rf/ϕ data for that pebble group. Contours at $m^* = 2.5$ (grey) and 1.5 (dark).

DISCUSSION OF RESULTS

We have treated Rf/ϕ data from rocks with an unknown deformation history and with inclusions of general 'ellipsoidal' shape with a method based on a theory valid for a rock containing parallel cylinders with elliptical cross section and subjected to a pure shear deformation. In the light of this, it becomes more important than usual to critically examine the produced results and apply, where possible, checks on their accuracy. Such checks are unfortunately not possible in the two cases discussed above because we have no independent way of estimating the whole-rock strains.

A striking feature of the results is the lack of precision in the determination of strain amount and viscosity ratio. This is particularly the case in the conglomerate example where, had it not been for the limits placed on the solution by the mutual consistency of the results, only a very broad range of values could have been quoted. We think this imprecision has two causes.

The first is related to the pre-strain eccentricity of the markers. The more equant the markers, the more imprecise the results will be. This can be illustrated with the aid of an extreme example; the case of deformed markers of originally circular shape. After deformation a group of such markers will show identical orientations and aspect ratios. These markers could be successfully de-strained in an infinitive number of ways, each corresponding to a different combination of viscosity contrast and strain ratio.

Clearly then the method relies on the presence of an initial shape factor. At the opposite extreme, a high initial aspect ratio of the markers should yield Rf/ϕ patterns which are insensitive to viscosity ratio. We anticipate this on the basis of the theory given by Bilby & Kolbuszewski (1977) from which can be deduced that the markers will become more 'Marchian' in their behaviour with increase in initial axial ratio. We predict such Rf/ϕ data will yield, on graphs like Figs. 2 and 5, elongate m^* contours almost parallel to the viscosity ratio axis. The strain ratio can be readily determined from such graphs, but not the viscosity ratio.

The second source of vagueness in the indicated strain and viscosity ratios has to do with assumed initial distribution of the markers' orientations. The assumed uniform distribution with its simple form means that the

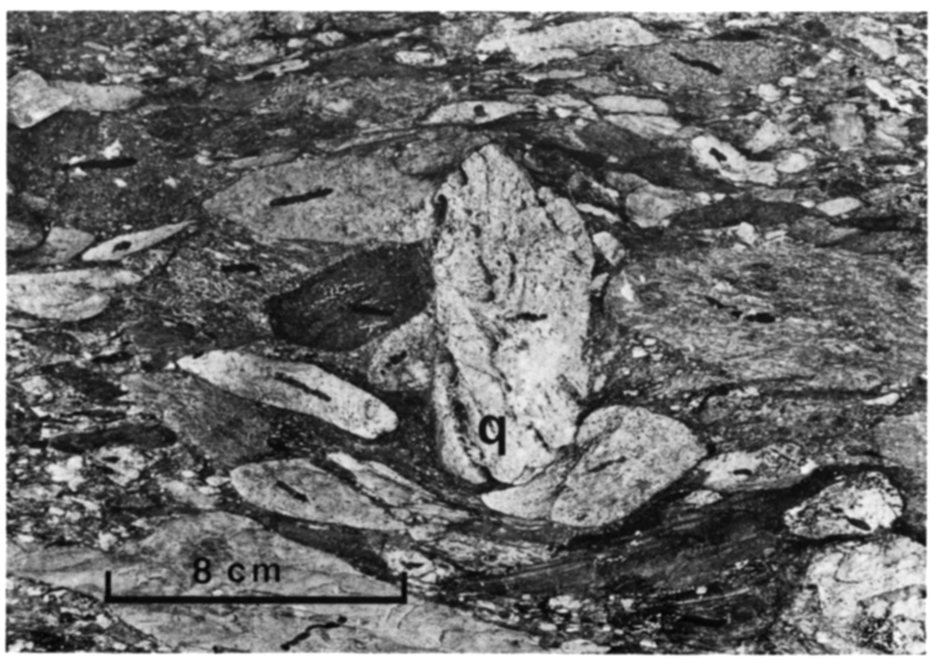

Fig. 4. Outcrop-view of deformed conglomerate illustrating the contrasting deformation of pebbles of different viscosities, Grythyttan area, Sweden. q, quartz pebble with its long axis perpendicular to the X-direction of finite strain.

relevant statistical tests are weak. This difficulty is not peculiar to the method here but is present in other Rf/ϕ methods where a uniform model is used for the pre-strain marker orientations. With regard to all Rf/ϕ analyses we feel it is more realistic to quote the full range of statistically acceptable solutions rather than simply the 'best-fit' result.

In spite of these drawbacks we feel the described method offers important advantages over other Rf/ϕ techniques which ignore the effects of viscosity contrasts. Although the amount of effort required to obtain the raw data is the same, the new method provides data on rheology of naturally deformed rocks; data which is difficult to obtain from other methods. Also, by considering the possible effects of competence contrast, this technique produces finite strain estimates which, in some cases (e.g. the quartz pebbles in Fig. 5), differ seriously from estimates based on the original Rf/ϕ methods, which make no allowance for competence differences.

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APPENDIX

The conglomerates come from the Swedish Grythyttan area where they occur at the sample location (coord. $66^{125} \times 14^{277}$) in thick beds with only relatively thin levels of finer clastics, dipping 70° to the west. The irregular, wavy and poorly developed cleavage is subvertical and strikes north. The conglomerates are pebble supported, with pebbles that very rarely exceed 30 cm in diameter. The fragments can easily be classified in the field as quartz (10–20%); leptite, hard and soft (30–40%); hälleflinta (15–30%) and slate, dark and light (25–40%). Axial ratios of the pebbles were measured on cleavage planes and *ac*-joints in natural outcrops of several square metres, and on the subhorizontal glaciated rock surface. The conglomerates form the younger part of the volcano-sedimentary 1.8–1.9 Ga Bergslagen Supracrustals (Oen *et al.* 1982). They contain the erosion products of the older part of this supracrustal series, and consist mainly of metavolcanics in the form of leptites that are fine-grained, and of hälleflintas that are extremely fine-grained quartzo-feldspathic rocks.

A number of thin sections of the pebbles indicates that the grain size of the quartz and feldspar in the groundmass of the soft leptite coincides with that of the hälleflinta, and that the grain size in the hard leptite is substantially more coarse. The slate proves to be rich in quartz of similar grain size to the hälleflintas.

These observations on grain size seem to substantiate the observation of Lisle & Savage (1983) that deformation increases with decreasing grain size, as the viscosity ratio calculated for the different pebble types increases with increasing grain size.